Neutron stars as sources of gravitational waves

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The global network of ground-based gravitational-wave detectors (the Advanced LIGO and the Advanced Virgo) is sensitive at the frequency range corresponding to relativistic stellar-mass compact objects. Among the promising types of gravitational-wave sources are binary systems and rotating, deformed neutron stars. I will describe these sources and present predictions of how their observations will, in the near future, contribute to modern astrophysics.

1 Introduction

Last two years were a turning-point for the gravitational-wave (GW) astrophysics. First direct detections of GWs from binary systems of massive stellar black holes with the Advanced LIGO detectors (Abbott et al. 2016 and subsequent ones) demonstrated the viability of the detection principle using the kilometer-size laser interferometry. This state-of-the-art (Advanced LIGO Aasi et al. 2015 and Advanced Virgo Acernese et al. 2015) technology creates an unprecedented opportunity for studying the Universe through a novel, never before explored channel of spacetime distortions (Einstein, 1916) measurements. For a mainstream astronomer, GW astronomy may seem a rather non-standard way of studying the Universe - it is more like ‘listening to’ than simply ’looking at’ the skies. By design motivated by the choice of potential sources, the ground-based GW detectors of LIGO and Virgo are sensitive to the range of frequencies similar to the audible range of human ears - between 10 Hz (limited by the seismic noise) and a few kHz (limited by the quantum nature of laser light). As in the case of an ear, a solitary laser interferometer is practically omnidirectional (has a poor angular resolution), and has no imaging capabilities; the localisation of sources is performed using a global network of detectors, currently realized by the three detectors of the LIGO-Virgo Collaboration.

GWs are created by a bulk movement of large, rapidly-moving masses: their accelerated movement provides a time-varying quadrupole, which is the lowest radiating moment in the general theory of relativity (GR). Once emitted, GWs are weakly coupled to the surrounding matter and propagate freely without scattering. This has to be contrasted with the electromagnetic emission that originates at the microscopic level, is strongly coupled to the surroundings and often reprocessed; it carries a reliable information from the last scattering surface only. GWs are therefore the perfect counterpart to the electromagnetic waves as they provide us with the information impossible to obtain by other means.

Science needed 100 years since the birth of GR (Einstein, 1915) to the moment when GWs are routinely directly registered by sophisticated apparatus on the Earth, and changing our understanding of the Universe. The Advanced LIGO and Advanced Virgo detectors were build thanks by part to the *indirect* evidence gathered in early times of the development of the theory and observations. In contrary to the Newton's theory of gravity, in which the state of moving masses does not evolve when no dissipation mechanisms are present, GR has the energy dissipation mechanism build-in into its very fabric. Early seminal works of prof. Andrzej Trautman convincingly showed that GWs carry the energy away from the radiating system, and thus are real physical phenomena and not artifacts of coordinate choice (Robinson & Trautman, 1960). From the observational side, prof. Bohdan Paczyński demonstrated that extremely short orbital periods of binaries containing white dwarfs, WZ Sge and HZ29, are possible only due to the emission of GWs (Paczyński, 1967). Full realization that binary systems do in fact evolve according to GR came with the discoveries of binary neutron stars systems in the 70s (for a summary see, e.g., Weisberg & Taylor 2005). Every binary system emits GWs, leading to the tightening of its orbit and increasing its orbital frequency, inevitably driving the components to merge. We may therefore expect that there exist an astrophysically interesting category of *cataclysmic* sources present in the sensitivity band of detectors only for a limited time: inspiralling and merging black-hole and neutron-star systems, as well the core-collapse supernovæ.

Neutron stars are among prime targets for the GW searches. They are less compact than black holes, but instead of being pure manifestation of the spacetime curvature they are composed of the most extreme matter existing currently in the Universe. Thus they provide truly unique conditions to study matter at the highest densities, pressures, in the presence of the most powerful magnetic fields and in the regime of strong gravity. These conditions cannot be reproduced (or even approximated) in the terrestrial laboratories. Neutron stars are involved in the most spectacular astrophysical phenomena like supernovæ, or magnetars' and gamma-ray bursts, yet very little is known about their internal microscopical composition, maximum masses and spins, radii, and other parameters.

2 Binary systems

I will first focus on the binary systems, since they were the first sources to be detected. I will use a simplified Newtonian description to describe the basic parameters one can infer from the inspiral part of the signal.

GW amplitude h (the spacetime distortion, "the strain") is proportional to $1/r$, with r being the luminosity (or rather 'loudness') distance to the source. This relation is a direct consequence of the conservation of energy. For a binary system of masses m_1 and m_2 at semi-major axis a, with the total mass $M = m_1 + m_2$, reduced mass $\mu = m_1 m_2/M$, and quadrupole moment $Q \propto \mu a^2$, h is proportional to the second time derivative of Q, representing the accelerated movement of masses: $h \propto \ddot{Q}/r \propto \mu a^2 \omega^2/r$. In more detail (Einstein, 1918), $h \simeq G^{5/3} \mu M^{2/3} \omega^{2/3}/(c^4 r)$, where we make use of Kepler's third law ($GM = a^3 \omega^2$).

Similarly, the GW luminosity $\mathcal{L}$ (the rate of GW-related energy loss, integrated over a sphere at a distance r) is $\mathcal{L} = dE_{GW}/dt \propto G h^2 \omega^2/c^5 \propto G \mu^2 a^4 \omega^6/c^5$ using the dimensional analysis arguments. Waves leave the system at the expense of its orbital energy $E_{orb} = -Gm_1m_2/(2a)$, yielding $dE_{orb}/dt \equiv Gm_1m_2\dot{a}/(2a^2) = -dE_{GW}/dt$. From the time derivative of the third Kepler's third law, $\dot{a} = -2a\dot{\omega}/(3\omega)$, we get the evolution of the orbital frequency driven by the gravitational-wave emission: $\dot{\omega} = (96/5)G^{5/3}\omega^{11/3}\mathcal{M}^{5/3}/c^5$.

The system changes by increasing its orbital frequency; at the same time the strain amplitude h of emitted waves also grows. This characteristic frequency-amplitude evolution is called the *chirp*, and the characteristic function of component masses $\mathcal{M} = \left(\mu^3M^2\right)^{1/5} = (m_1m_2)^{3/5}/(m_1+m_2)^{1/5}$ is called the *chirp mass*. Orbital frequency is related in a straightforward manner to the GW frequency f_{GW}: from the geometry of the problem it is evident that the frequency of radiation is predominantly at twice the orbital frequency, $f_{GW} = \omega/\pi$. The chirp mass $\mathcal{M}$ is therefore directly measured by the detector registering the evolution of f_{GW}; may be e.g., recovered the time-frequency spectrograms:

$$\mathcal{M} = \frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3}f_{GW}^{-11/3}\dot{f}_{GW}\right)^{3/5}. \tag{1}$$

Time evolution of $\dot{f}_{GW}$ and h give also the distance to the source r. Again, since it is a function of the amplitude and frequency parameters, the loudness distance r is a directly measurable quantity:

$$r = \frac{5}{96\pi^2}\frac{c}{h}\frac{\dot{f}_{GW}}{f_{GW}^3}. \tag{2}$$

These properties make the merging binary systems analogues to standard candles of traditional astronomy, hence they are sometimes called "standard sirens" (Holz & Hughes, 2005). The idea of using well-understood signals to infer the distance and constrain the cosmological parameters was proposed first by Schutz (1986).

The GW signal is extracted from the noisy data using matched-filter techniques, correlating the signal model with the data as it evolves in the sensitivity band, which means that following the phase of the signal is crucial in the process. For wide binary systems, point particles or black holes the frequency-domain phase Ψ may be expanded in the following series of a small parameter:

$$\Psi(f_{GW}) \equiv \Psi_{PP}(f_{GW}) \propto \frac{3M}{128\mu v^{5/2}}\sum_{k=0}^{N}\alpha_k v^{k/2}, \tag{3}$$

that in this example is the orbital velocity $v \propto (\pi\mathcal{M}f_{GW})^{1/3}$. At cosmological distances, the observed frequency f_{GW} is redshifted by the expansion of the Universe by $(1+z)$ (that is, $f_{GW} \to f_{GW}/(1+z)$). Purely vacuum GR is not equipped with the in-build mass scale, so if the source doesn't emit light (like in the case of black holes), there is no way of breaking the redshift degeneracy between the f_{GW} and $\mathcal{M}$.

Fortunately, in case of binary neutron stars which are without a doubt material sources, the point-particle description breaks down sufficiently early before the merger for additional effects related to interactions between the bodies could be detected. Early part of the inspiral is dominated by gravitational back reaction (a function of component masses and spins). In the late inspiral however, tidal effects (mutual deformation of stars in the gravitational field of the companion) become important. A relation between the tidal tensor $\mathcal{E}_{ij}$ of one of the components inducing quadrupole moment Q_{ij} in the other is, in the adiabatic approximation, $Q_{ij} = \lambda(EOS)\mathcal{E}_{ij}$, where the *tidal deformability* λ is a function of the equation of state (EOS) and the mass of the star, $\lambda = (2/3)k_2(EOS)R^5$. The quantity k_2 is the Love number (Love, 1911), whereas R is the radius of the star. From the scaling one can deduce the deformability is a high order effect (5th post-Newtonian order in Eq. 3, $k = 10$), so it becomes large enough only for very tight, relativistic binary systems a few orbits before the merger. Measurement of the components' λs can reveal, in addition to already measured masses, the radius and compressibility of the star. Moreover, the fact that the components deform each other during inspiral changes the behavior of the phase of the GW signal: $\Psi \neq \Psi_{PP}$ for sufficiently tight systems, and must be complemented by an additional term related to the tidal deformation, Ψ_{tidal}. This additional term is a function of the Love numbers, masses and radii of stars, hence it breaks the degeneracy in the expansion in the small parameter v in Eq. 3. In principle, the measurement of tidal terms allows at the same time to measure the loudness distance and establish the cosmological redshift.

3 Persistent gravitational radiation

Second category are the *continuous* sources, e.g., very wide binary systems, or rotating, deformed or oscillating neutron stars ("GW pulsars"). These sources produced long-lived GWs (duration T is much longer than the observing time T_{obs}) and are nearly periodic, with the GW frequency usually proportional to the characteristic rotational frequency of the object, $f_{GW} \propto f_{rot}$. Mechanisms responsible for creating a time-varying quadrupole in this type of sources are related to deformations sustained by elastic and/or magnetic stresses ("mountains", $f_{GW} = 2f_{rot}$), unstable Rosby modes driven by the Coriolis force ($f_{GW} = 4/3f_{rot}$), free precession ($f_{GW} \propto f_{rot} + f_{prec}$) or accretion contributing to deformation via thermal gradients ($f_{GW} \simeq f_{rot}$) (for a recent overview of the modeling of periodic GWs see Lasky 2015). GW amplitude (strain) h_0 for these sources is proportional to the degree of asymmetry and the spin frequency, $h_0 \propto I_3\epsilon f^2$, where $\epsilon = (I_1 - I_2)/I_3$ is the deformation, and I_i are the value of the moment of inertia along the principal axes, I_3 being aligned with the rotation axis. Depending on the dense-matter models, $\epsilon_{max} = 10^{-3} - 10^{-6}$. Recent results on the LIGO O1 data Abbott et al. (2017b,c) did not provide direct detections (yet), but physically interesting upper limits were obtained for several known sources and across a wide frequency range. The upper limits for GW radiation are compared with the available reservoir of energy, rotational energy E_{rot}. Rotational energy loss $\dot{E}_{rot} \propto f\dot{f}$, and the energy emitted in GWs scales like $\dot{E}_{GW} \propto f^6 I_3^2 \epsilon^2$. Assuming $\dot{E}_{rot} = \dot{E}_{GW}$ and the knowledge of I_3 and the distance to the source, we get the spin-down upper limit for the strain, $h_0^{sd} \propto \sqrt{|\dot{f}|I_3/f}$. Recent results show that the Crab pulsar emits less than 2×10^{-3} $\dot{E}_{rot}$ in GWs, and the Vela pulsar less than 10^{-2} $\dot{E}_{rot}$. These upper limits improve previously obtained by a factor of 2.5 and allow for excluding extreme radiation models (for more details on the status of the data analysis of continuous GWs from rotating neutron stars see Bejger 2017).

Persistent GWs are also expected in the form of stochastic background produced by the populations of (continuous or transient) sources, or even by GWs created in the very early Universe.

4 The dawn of multi-messenger astronomy

GWs and photons provide complementary information about the physics of the source and its environment. From GWs we may learn about the masses, spins and eccentricity in case of binary systems, and deformation in case of rotating objects, system orientation, distance, and also the size of the population and the rate at which astrophysical phenomena happen. Electromagnetic observations provide precise sky localisation, redshifts of the host galaxies, information about the local environment, emission processes and acceleration mechanisms. Global network of three interferometers of LIGO and Virgo is currently able to provide sky localisation within ~ 10 square degrees from the GW observations alone (triangulation) which allow for rapid electromagnetic follow-up. At its nominal sensitivity, the LIGO-Virgo collaboration will be able to routinely detect binary black hole mergers up to 1 Gpc distance, and neutron-star mergers up to 200 Mpc.

5 Afterword: GW170817

Judging from the lively discussion during and after my talk, the audience saw through my slides and was already aware of the spreading rumor of a new breakthrough discovery, that became public approximately one month later, on October 16, 2017. Indeed, August 2017 was an extremely interesting experience: first binary black-hole merger observation with the global network of three detectors of Advanced LIGO and the Advanced Virgo (Abbott et al., 2017e), as well as the first Advanced LIGO and Advanced Virgo detection of a nearby (at a distance of 40 Mpc) binary neutron-star merger (Abbott et al., 2017f), followed by a short gamma-ray burst (Abbott et al., 2017d) and broad-band electromagnetic emission observational campaign (Abbott et al., 2017g) and the kilonova study. The detection of a beautiful, surprisingly strong chirp signal from the closest short gamma-ray burst to date served as the best proof for theoretical ideas put forward by prof. Paczyński: connection between short gamma-ray bursts and neutron-star mergers (Paczynski, 1986) and the physics of kilonova (Li & Paczyński, 1998). GW170817 was the best demonstration of techniques described in this talk: precise triangulation with three GW detectors crucial for the kilonova follow-up, direct „standard candle" (i.e., bypassing traditional „distance ladders") measurement of distance to the host galaxy and independent measurement of the Hubble constant (Abbott et al., 2017a), measurement of the speed of gravitational waves, and tidal deformabilities of component neutron stars. Thanks to neutron stars we now witness a true beginning of the GW astronomy.

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